S Parameter in the Holographic Walking/Conformal Technicolor

Kazumoto Haba,[1,*] **Shinya Matsuzaki**,[2,†] and **Koichi Yamawaki**[1,‡]

[1] *Department of Physics, Nagoya University, Nagoya, 464-8602, Japan.*
[2] *Department of Physics and Astronomy, University of North Carolina, Chapel Hill 27599-3255.*

We explicitly calculate the S parameter in entire parameter space of the holographic walking/conformal technicolor (W/C TC), based on the deformation of the holographic QCD by varying the anomalous dimension from $\gamma_m \simeq 0$ through $\gamma_m \simeq 1$ continuously. The S parameter is given as a positive monotonic function of ξ which is fairly insensitive to γ_m and continuously vanishes as $S \sim \xi^2 \to 0$ when $\xi \to 0$, where ξ is the vacuum expectation value of the bulk scalar field at the infrared boundary of the 5th dimension $z = z_m$ and is related to the mass of (techni-) ρ meson (M_ρ) and the decay constant (f_π) as $\xi \sim f_\pi z_m \sim f_\pi/M_\rho$ for $\xi \ll 1$. However, although ξ is related to the techni-fermion condensate $\langle \bar{T}T \rangle$, we find no particular suppression of ξ and hence of S due to large γ_m, based on the correct identification of the renormalization-point dependence of $\langle \bar{T}T \rangle$ in contrast to the literature. Then we argue possible behaviors of f_π/M_ρ as $\langle \bar{T}T \rangle \to 0$ near the conformal window characterized by the Banks-Zaks infrared fixed point in more explicit dynamics with $\gamma_m \simeq 1$. It is a curious coincidence that the result from ladder Schwinger-Dyson and Bethe-Salpeter equations well fits in the parameter space obtained in this paper. When $f_\pi/M_\rho \to 0$ is realized, the holography suggests a novel possibility that f_π vanishes much faster than the dynamical mass m does.

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I. INTRODUCTION

The origin of the electroweak symmetry breaking is the most urgent issue to be resolved at the LHC experiments. In the standard model, Higgs boson is introduced just as a phenomenological input only for the sake of making particles acquire masses. As such the standard model does not explain the origin of the electroweak symmetry breaking. With the existence of Higgs boson as an elementary particle, moreover, one necessarily faces with some problems such as naturalness, etc.

One of the candidates which resolve these problems is the technicolor (TC) model [1]. In the framework of TC, the origin of the electroweak symmetry breaking is explained dynamically without introduction of Higgs boson. The simplest model of TC, just a simple scale-up of QCD, however, does not pass the electroweak precision test, especially, suffers from a large contribution of $\mathcal{O}(1)$ to the Peskin-Takeuchi S parameter [2], while the electroweak precision test shows that the value of the S parameter is less than about 0.1.

There is an interesting possibility [3, 4] that contributions to the S parameter can be reduced in the case of the walking/conformal TC (W/C TC) [5, 6, 7, 8], initially dubbed as "scale-invariant TC" [6], with almost non-running (walking) gauge coupling near the conformal fixed point, which produces a large anomalous dimension $\gamma_m \simeq 1$ of the techni-fermion condensate operator $\bar{T}T$ [6] (for reviews, see [9]). A salient feature of this theory is the appearance of a composite Higgs boson as a (massive) techni-dilaton [6] associated with the spontaneous breaking of the scale invariance (as well as the explicit breaking due to the scale anomaly) . A typical example [10, 11] of such a W/C TC is based on the Banks-Zaks infrared fixed point [12] (BZ-IRFP) α_* in the large N_f QCD, QCD with many massless flavors $N_f \gg 3$ ($N_f < 11N_c/2$)). Looking at the region $0 < \alpha < \alpha_*$, we note that $\alpha_* \searrow 0$ when $N_f \nearrow 11N_c/2$, and hence there exists a certain region ($N_f^* <$) $N_f^{\rm cr} < N_f < 11N_c/2$ ("conformal window") such that $\alpha_* < \alpha^{\rm crit}$, where $\alpha^{\rm crit}$ is the critical coupling for the spontaneous chiral symmetry breaking and hence the chiral symmetry gets restored in this region. Here $\alpha^{\rm crit}$ may be evaluated as $\alpha^{\rm crit} = \pi/3C_2(F)$ in the ladder approximation, in which case we have $N_f^{\rm cr} \simeq 4N_c$ [10] #1 #2. Related to the conformal symmetry, this phase transition ("conformal phase transition"[11])

*haba@eken.phys.nagoya-u.ac.jp
†synya@physics.unc.edu
‡yamawaki@eken.phys.nagoya-u.ac.jp

#1 In the case of $N_c = 3$, this value $N_f^{\rm cr} \simeq 4N_c = 12$ is somewhat different from the lattice value [13] $6 < N_f^{\rm cr} < 7$, but is consistent with more recent lattice results [14].

#2 There is another possibility for the W/C TC with much less N_f based on the higher TC representation [15], although explicit ETC

has unusual nature that the order parameter changes continuously but the spectrum does discontinuously at the phase transition point.

When it is applied to TC, we set α_* slightly larger than $\alpha^{\rm crit}$ (slightly outside of the conformal window), with the running coupling becoming larger than the critical coupling only in the infrared region, we have a condensate or the dynamical mass of the techni-fermion m of the order of such an infrared scale which is much smaller than the intrinsic scale of the theory $\Lambda_{\rm TC}(\gg m)$. Although the BZ-IRFP actually disappears due to decoupling of massive fermion at the scale of m, the coupling is still walking due to the remnant of the BZ-IRFP in a wide region $m < \mu < \Lambda_{\rm TC}$. Then the theory develops a large anomalous dimension $\gamma_m \simeq 1$ and enhanced condensate $\langle \bar{T}T\rangle|_{\Lambda_{\rm TC}} \sim \Lambda_{\rm TC} m^2$ at the scale of $\Lambda_{\rm TC}$ which is usually identified with the ETC scale $\Lambda_{\rm TC} = \Lambda_{\rm ETC}$ [10, 11]. Note that $m \to 0$ as $\alpha_* \searrow \alpha^{\rm crit}$ and the mass of techni-dilaton, $M_{\rm TD} \simeq \sqrt{2}m$, #3 also vanishes to be degenerate with the Nambu-Goldstone (NG) boson, although there is no light spectrum in the conformal window as a characteristic feature of the conformal phase transition.

The W/C TC, however, has a calculability problem, since its non-perturbative dynamics is not QCD-like at all, and hence no simple scaling of QCD results would be available. The best thing we could do so far has been a straightforward calculation based on the SD equation and (inhomogeneous) BS equation in the ladder approximation [4], which is however not a systematic approximation and is not very reliable in the quantitative sense.

Of a late fashion, based on the so-called AdS/CFT correspondence, a duality of the string in the anti-de Sitter space background-conformal field theory [18], holography gives us a new method which may resolve the calculability problem of strongly coupled gauge theories [19]: Use of the holographic correspondence enables us to calculate Green functions in a four-dimensional strongly coupled theory from a five-dimensional weakly coupled theory. For instance, QCD can be reformulated based on the holographic correspondence either in the bottom-up approach [20, 21] or in the top-down approach [22]. In both approaches we end up with the five-dimensional gauge theory for the flavor symmetry, whose infinite tower of Kaluza-Klein modes describe nicely a set of the massive vector/axialvector mesons as the gauge bosons of Hidden Local Symmetries (HLS) [23, 24, 25], or equivalently as the Moose [26]. Although a holographic description is valid only for large N_c limit, several observables of QCD have been reproduced within 30 % errors in both approaches. Moreover, through the high-energy behavior of current correlators in operator product expansion, some consistency with the QCD has been confirmed in the bottom-up approach.

Recently several authors [27, 28] calculated the S parameter in the W/C TC as an application of the above technique of bottom-up holographic QCD [20, 21] to the holographic W/C TC: They made some deformation adjusting a profile of a 5-dimensional bulk scalar field which is related to the anomalous dimension of techni-fermion condensate γ_m. They claimed that when $\gamma_m \equiv 1$, the S parameter for certain parameter choices is substantially reduced compared to that of the QCD-like theory with $\gamma_m \simeq 0$. It is not clear, however, how the non-trivial feature of the dynamics of walking/conformal theory contributes to that reduction, since they discuss only specific parameter choices relevant to specific TC models. Actually, *it is not* $\gamma_m \equiv 1$ *but* $\gamma_m \simeq 1$ ($\gamma_m < 1$) that is needed for realistic model building of W/C TC where $\gamma_m = 1$ should be regarded an idealized limit of $\gamma_m \to 1$ from the side of $\gamma_m < 1$.

In this paper, based on the holographic correspondence in the bottom-up approach, we calculate the S parameter in the W/C TC, treating the anomalous dimension γ_m as a free parameter as $0 < \gamma_m < 1$, varying *continuously* from the QCD monitor value $\gamma_m \simeq 0$ through the one of the W/C TC $\gamma_m \simeq 1$. We calculate S as an explicit function of $\xi \equiv Lv(z)\Big|_{z=z_m}$ in the entire region of ξ, where L is the AdS$_5$ radius, $v(z) \equiv \langle \Phi(x^\mu, z)\rangle$ is the vacuum expectation value of the bulk scalar field $\Phi(x^\mu, z)$ ($\mu = 0, 1, 2, 3$) and the 5th dimension z has both infrared cutoff z_m and ultraviolet cutoff ϵ: $\epsilon < z < z_m$. This is in contrast to the previous authors [27, 28] whose discussions correspond to specific values of the parameter ξ and are restricted to the case of $\gamma_m \equiv 1$ as the W/C TC. Since the realistic model building of W/C TC is *not* for $\gamma_m \equiv 1$ but $\gamma_m \simeq 1$ (($\gamma_m < 1$) , the analysis of Ref. [27, 28] could be a too much idealization, unless their result is continuously connected with the limit $\gamma_m \to 1$ from the side of $0 < \gamma_m < 1$. Actually, it turns out that the analysis of Ref. [27] is *not* continuously connected with the $\gamma_m \to 1$ limit of our result and thus would not precisely correspond to the realistic situation of the W/C TC we are interested in.

Then we find that S is a positive function of ξ in accord with the previous authors [27, 29]. We also find that both $z_m f_\pi$ and S are monotonically increasing functions of ξ which is related to chiral condensate $\langle \bar{T}T\rangle$. Noting that $z_m^{-1} \sim M_\rho$ with M_ρ being the techni-ρ meson mass, we have an expression of S as a function of f_π/M_ρ.

Most remarkably, we find that S *continuously goes to zero* as $S \sim \xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \to 0$ if $\xi \to 0$ (See Fig. 3),

model building would be somewhat involved.

#3 This estimate [16] is based on the ladder Schwinger-Dyson (SD) equation for the gauged Nambu-Jona-Lasinio model which well simulates [10, 11] the conformal phase transition in the large N_f QCD. Actually, the result is consistent with the straightforward calculation [17] of scalar bound state mass, $M_{\rm TD} \sim 1.5\, m$, through coupled use of the SD equation and (homogeneous) Bethe-Salpeter (BS) equation in the ladder approximation.

which is also in accord with the previous author [28] discussing the case of $\gamma_m \equiv 1$. We also find that the result is fairly insensitive to the value of γ_m unless we have substantial reduction of ξ or f_π/M_ρ for larger γ_m: Writing $\hat{S} = S/(N_f/2) = B f_\pi^2/M_\rho^2$, we find $B \simeq 27$ for $\gamma_m \simeq 1$ and $B \simeq 32$ for $\gamma_m \simeq 0$ (See Fig. 5). Although the result of (slight) decreasing tendency for fixed f_π/M_ρ is not inconsistent with Ref. [27], their value for $\gamma_m \equiv 1$ is somewhat smaller than ours with $\gamma_m \to 1$.

Our result roughly coincides with the well-known fact (see e.g. Ref. [25]) that $\hat{S}$ in QCD case is given by the ρ meson dominance as $\hat{S} = -16\pi L_{10} = 4\pi \cdot (F_\rho/M_\rho)^2 = 4\pi a (f_\pi/M_\rho)^2 = (g^2/(4\pi))^{-1}$, with $a \simeq 2$ experimentally, where $F_\rho = \sqrt{a} f_\pi$ is the decay constant of the ρ meson (or, of the fictitious Nambu-Goldstone boson absorbed into the ρ meson in the language of HLS) and g the gauge coupling of HLS. However, our result is highly nontrivial, since the holography includes an infinite tower of the vector and axialvector meson poles not just the lowest ρ pole contribution. Note that the contributions of vector mesons are opposite in sign to those of the axialvector mesons and therefore the infinite sum of all contributions could in principle result in any functional form such as giving a non-vanishing constant in the limit $f_\pi/M_\rho \to 0$.

It thus opens a novel possibility for having small S parameter, if we find a mechanism of suppressing $\xi^2 N_{\rm TC} \sim f_\pi/M_\rho \ll 1$ particularly near the conformal phase transition $m \to 0$. Then the next issue is whether or not we can realize $\xi^2 N_{\rm TC} \sim f_\pi/M_\rho \ll 1$ in the W/C TC.

It is also to be noted that the above continuous vanishing of S in the case that $\xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \to 0$ is highly nontrivial in sharp contrast to the usual perturbative calculation where the S parameter does not vanish even in the chiral restoration limit $m \to 0$, i.e., $S \to \frac{N_f}{2}\frac{N_{\rm TC}}{6\pi}$ for $N_{\rm TC}$ technicolors and N_f techni-flavors, although it is identically zero when $m = 0$ as it should, i.e. there is a discontinuity at the chiral phase transition.

Unfortunately, the holographic approach as it stands cannot decide whether or not $\xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \to 0$: f_π/M_ρ is given by a certain function of ξ and γ_m which are both arbitrary parameters in this approach. Although ξ is related with the chiral condensate $\langle \bar{T}T \rangle$ which vanishes at the conformal phase transition point, we find no direct suppression of ξ or f_π/M_ρ and hence of S due to the large γ_m in contrast to the previous authors [27, 28]. Based on the correct identification of the renormalization scale of $\langle \bar{T}T \rangle$, we have $\xi \sim (m z_m)^{3-\gamma_m} \sim (m/M_\rho)^{3-\gamma_m}$, independently of the non-physical renormalization point L or ϵ, which may or may not be small even if $\langle \bar{T}T \rangle \to 0$, unless we know $m/M_\rho \ll 1$. Thus ξ is not necessarily a small parameter in this framework even for $m \to 0$, not to mention for $L \to 0$, $\epsilon \to 0$. Then the only possibility to realize $\xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \to 0$ would be to discuss more concrete dynamics approaching the conformal phase transition where we have $m \to 0$ (or $f_\pi \to 0$) not just a large anomalous dimension $\gamma_m \simeq 1$. Actually the effects of anomalous dimension are highly involved, combined with the scaling of f_π/M_ρ as $m \to 0$, as seen in the direct calculation based on the ladder SD and BS equations [4].

We then discuss possible scaling behavior of $\xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \to 0$ near the conformal window $m \to 0$. Although a simple large N_c argument would always imply $\hat{S} \sim \xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \sim N_{\rm TC} \to \infty$, the conformal phase transition takes place due to the Banks-Zaks infrared fixed point which only can be realized for large N_f with $N_c/N_f =$ fixed. Then the behavior of $(f_\pi/M_\rho)^2$ near the conformal phase transition is highly nontrivial. Obviously three possibilities in the limit of $m \to 0$: i) $f_\pi/M_\rho \to \infty$, ii) $f_\pi/M_\rho \to \text{constant} \neq 0$, iii) $f_\pi/M_\rho \to 0$.

We find that the case i) is realized only for $\xi \gg 1$, since f_π/M_ρ is the monotonically increasing function of ξ. In this case we have $\frac{f_\pi}{\sqrt{N_{\rm TC}}} \sim m$, which is the familiar scaling relation realized in QCD. Actually the case i) corresponds to the Vector Manifestation proposed in the HLS loop calculation [25, 30].

The case ii) where $f_\pi/M_\rho \to \text{constant} \neq 0$ is realized only for the case $\xi \sim (m/M_\rho)^{3-\gamma_m} \to \text{constant} \neq 0$ and hence $S \to \text{constant} \neq 0$ for $m \to 0$. In this case $\frac{f_\pi}{\sqrt{N_{\rm TC}}} \sim M_\rho \sim m$, which is the same scaling relation as the case i). The case ii) actually corresponds to the straightforward calculation based on the ladder SD and BS equations [4, 17]. It is amusing that a set of $(\xi, \hat{S}/N_{\rm TC})$, ξ obtained from homogeneous BS equation and $\hat{S}/N_{\rm TC}$ from inhomogeneous BS equation both combined with SD equation, well coincides with a single point on the line of the $(\xi, \hat{S}/N_{\rm TC})$-plane obtained in this paper.

The most interesting case for the TC is case iii) in which we have $f_\pi/M_\rho \to 0$ as $m \to 0$. We find that the case iii) is realized only for $\xi \ll 1$, since f_π/M_ρ is a monotonically increasing function of ξ, although we have no explicit dynamics at this moment. We shall discuss some possible dynamics for this case which will be tested by future studies. In the case iii) we find a novel scaling property of f_π vanishing much faster than m near the conformal window, resulting in the form $\frac{f_\pi}{\sqrt{N_{\rm TC}}} \sim m (m/M_\rho)^{2-\gamma_m}$, which is quite different from the familiar one $\frac{f_\pi}{\sqrt{N_{\rm TC}}} \sim m$. This could be testable by lattice calculation for large N_f QCD.

Although the bottom-up approach of the holography does not explicitly uses the large $N_{\rm TC}$, the top-down approach needs that limit. Then the result here might be potentially valid only for large $N_{\rm TC}$ not near the conformal phase transition region where N_f is large with $N_f/N_{\rm TC} =$ fixed. Nevertheless, the result of this paper $\hat{S} \sim (f_\pi/M_\rho)^2$ might be valid beyond the leading order of $1/N_{\rm TC}$. Then it would be highly desired to investigate the possibility for $f_\pi/M_\rho \to 0$ in some explicit dynamics.

The paper is organized as follows:

In Sec. II we briefly review the framework of calculations in the holographic W/C TC model of Refs. [27, 28] based on the bottom-up holographic QCD [20, 21].

In Sec. III we calculate the S parameter in models holographically dual to W/C TC allowing for varying values of the large anomalous dimension of techni-fermion condensation γ_m from the QCD monitor value $\gamma_m \simeq 0$ to the W/C TC value $\gamma_m \simeq 1$.

In Sec. IV we identify the renormalization-point of the $\langle \bar{T}T \rangle$, based on which we find that there is no suppression factor solely due to large γ_m.

In Sec. V we classify holographic W/C TC models into three cases, i) $f_\pi/M_\rho \to \infty$, ii) $f_\pi/M_\rho \to$ constant $\neq 0$, iii) $f_\pi/M_\rho \to 0$ as $m \to 0$ near the conformal window arising due to the Banks-Zaks infrared fixed point with $\gamma_m \simeq 1$. As an explicit dynamics for the case ii) we find a curious coincidence of the result of ladder SD and BS equations with the result in this paper. It is also shown that if the case iii) is realized, the S parameter goes to zero at the edge of the conformal window in such a way that $f_\pi \to 0$ scales as $m \to 0$ much faster than the familiar form, $f_\pi^2 \sim m^2$.

Sec. VI is devoted to summary and discussion.

In Appendix A we discuss subtlety of the limit $\gamma_m \to 1$ and $\gamma_m = 1$.

In Appendix B we discuss the Pagels-Stokar formula in comparison with the holographic result.

II. REVIEW OF HOLOGRAPHIC CALCULATIONS

In this section we briefly review the framework of calculations in the holographic W/C TC model of Refs. [27, 28] with $\gamma_m = 1$ which is the deformation of the the bottom-up holographic QCD [20, 21] with $\gamma_m = 0$ by adjusting a profile of a 5-dimensional bulk scalar field. Here we consider a generic case with $0 < \gamma_m < 1$.

A holographic model [20, 21, 27, 28] is defined on the 5-dimensional anti-de Sitter space (AdS_5) with the metric,

$$ds^2 = g_{MN} dx^M dx^N = \left(\frac{L}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right), \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{diag}[1, -1, -1, -1]$ is a metric on 4-dimensional space-time spanned by the coordinate x_μ, and L denotes the curvature radius of AdS_5. The fifth direction z is compactified on the interval,

$$\epsilon \leq z \leq z_m \,. \tag{2}$$

A holographic action [20, 21] possessing an $SU(N_f)_L \times SU(N_f)_R$ gauge symmetry in 5 dimensions is constructed from $SU(N_f)_{L,R}$ gauge fields $L_M(x,z)$ and $R_M(x,z)$, and a scalar field $\Phi(x,z)$ transforming under the $SU(N_f)_L \times SU(N_f)_R$ gauge symmetry as a bi-fundamental representation. The action is given by [#4],

$$\begin{aligned} S_5 \;=\; & \frac{1}{g_5^2} \int d^4x \int_\epsilon^{z_m} dz\, \sqrt{g} \\ & \times \left(-\frac{1}{2} \mathrm{Tr}\left[L_{MN} L^{MN} + R_{MN} R^{MN} \right] + \mathrm{Tr}\left[D_M \Phi^\dagger D^M \Phi - m_5^2 \Phi^\dagger \Phi \right] \right), \end{aligned} \tag{3}$$

where g_5 denotes the gauge coupling in 5 dimensions and $g = \det[g_{MN}] = (L/z)^{10}$. The covariant derivative acting on the scalar field Φ is defined as

$$D_M \Phi = \partial_M \Phi + i L_M \Phi - i \Phi R_M \,. \tag{4}$$

This Φ may be parametrized by using scalar and pseudo-scalar fields, ϕ and P, as

$$\Phi(x,z) = \phi(x,z) \exp[i P(x,z)/v(z)] \,, \tag{5}$$

with $v(z) = \frac{1}{\sqrt{2}} \langle \phi \rangle$ being the vacuum expectation value (VEV) of Φ.

For later convenience, we introduce 5-dimensional vector and axialvector gauge fields V_M and A_M defined by

$$V_M = \sqrt{\frac{1}{2}} (L_M + R_M) \,, \qquad A_M = \sqrt{\frac{1}{2}} (L_M - R_M) \,, \tag{6}$$

[#4] Here $\mathrm{Tr}[T^a T^b] = \frac{1}{2} \delta^{ab}$ and $L(R)_{MN} = \partial_M L(R)_N - \partial_N L(R)_M - i[L(R)_M, L(R)_N]$.

and we choose a gauge,

$$V_z(x,z) = A_z(x,z) \equiv 0\,. \tag{7}$$

Based on AdS/CFT correspondence, boundary conditions for the bulk fields V_M, A_M, and Φ are chosen so that their UV boundary values are related to the external sources in TC theories in the limit of $\epsilon \to 0$: For the VEV of Φ, the UV boundary value is related to the external source for the techni-fermion condensate $\langle \bar{T}T \rangle$, namely, the current mass of techni-fermion M in such a way that

$$\begin{aligned} M &\equiv \lim_{\epsilon\to 0} \mathcal{M}, \\ \mathcal{M} &= \left(\frac{L}{\epsilon}\right)^{\gamma_m} \left(\frac{L}{\epsilon} v(z)\right)\Big|_{z=\epsilon}, \end{aligned} \tag{8}$$

where γ_m stands for the anomalous dimension of the techni-fermion condensate $\langle \bar{T}T \rangle$. The AdS/CFT correspondence makes it possible to associate m_5, the mass of the scalar field Φ, with the anomalous dimension γ_m:

$$m_5^2 = -\frac{(3-\gamma_m)(\gamma_m+1)}{L^2}\,. \tag{9}$$

We introduce the variable ξ for the IR boundary value of VEV of Φ,

$$\xi = Lv(z)\Big|_{z=z_m}\,, \tag{10}$$

which corresponds to $\langle \bar{T}T \rangle$ as will be seen later (Eq.(27)).

We shall later discuss M and the corresponding $\langle \bar{T}T \rangle$ are quantities renormalized at the scale $1/L$ (see Sec.IV A), whereas ξ is the quantity renormalized at $1/z_m$.

As for the bulk gauge fields V_μ and A_μ, the UV boundary values play the role of the external sources (v_μ, a_μ) for the vector and axialvector currents coupled to the holographic TC. Accordingly, under $V_z \equiv A_z \equiv 0$ gauge, the boundary condition may be chosen,

$$\begin{aligned} &\partial_z V_\mu(x,z)\Big|_{z=z_m} = \partial_z A_\mu(x,z)\Big|_{z=z_m} = 0\,, \\ &V_\mu(x,z)\Big|_{z=\epsilon} = v_\mu(x)\,, \qquad A_\mu(x,z)\Big|_{z=\epsilon} = a_\mu(x)\,. \end{aligned} \tag{11}$$

With these boundary conditions (10) and (11), the equations of motion for the bulk gauge fields are completely solved at the classical level. By substituting those solutions into the action (3), the effective action W is expressed as a functional of the UV boundary values/external sources, $\mathcal{M}$, v_μ, and a_μ, i.e., $W = W[\mathcal{M}, v_\mu, a_\mu]$. The two-point Green functions are then readily calculated as

$$\left.\frac{\delta^2 W[v_\mu]}{\delta\tilde{v}^a_\mu(q)\delta\tilde{v}^b_\nu(0)}\right|_{v_\mu=0} = i\int_x e^{iq\cdot x}\langle J_V^{a\mu}(x)J_V^{b\nu}(0)\rangle = -\delta^{ab}\left(g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_V(-q^2)\,, \tag{12}$$

$$\left.\frac{\delta^2 W[a_\mu]}{\delta\tilde{a}^a_\mu(q)\delta\tilde{a}^b_\nu(0)}\right|_{a_\mu=0} = i\int_x e^{iq\cdot x}\langle J_A^{a\mu}(x)J_A^{b\nu}(0)\rangle = -\delta^{ab}\left(g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_A(-q^2)\,, \tag{13}$$

$$\lim_{\epsilon\to 0} i\left.\frac{i\delta W[\mathcal{M}]}{\delta \mathcal{M}}\right|_{\mathcal{M}=0} \equiv \lim_{\epsilon\to 0} i\left.\frac{i\delta W[M]}{\delta M}\right|_{M=0} = \langle \bar{T}T \rangle\,, \tag{14}$$

where $\tilde{v}^\mu(q)$ and $\tilde{a}^\mu(q)$ respectively denote the Fourier component of $v^\mu(x)$ and $a^\mu(x)$.

Once the current correlators are calculated, we can compute the S parameter. We define $\hat{S}$ as the S parameter per each techni-fermion doublet,

$$\hat{S} = \frac{S}{N_f/2}\,, \tag{15}$$

which is expressed by the vector and axialvector current correlators Π_V and Π_A as

$$\hat{S} = -4\pi \frac{d}{dQ^2}\left[\Pi_V(Q^2) - \Pi_A(Q^2)\right]_{Q^2=0}\,, \tag{16}$$

where $Q \equiv \sqrt{-q^2}$. In the next subsections we shall calculate those current correlators, $\langle \bar{T}T \rangle$, Π_V and Π_A.

A. Generating Functional $W[\mathcal{M}]$ and $\langle \bar{T}T \rangle$

Let us focus on a portion of the action (3) relevant for the VEV of $\Phi(x,z)$, $\langle \phi \rangle = v(z)$:

$$S_5 \mid_v = \int d^4x \int_\epsilon^{z_m} dz \, \frac{L^3}{2g_5^2} \mathrm{Tr} \left[-\frac{1}{z^3} (\partial_z v(z))^2 + \frac{(3-\gamma_m)(1+\gamma_m)}{z^5} v^2(z) \right] , \tag{17}$$

which leads to the following classical equation of motion for $v(z)$:

$$\partial_z \left(\frac{1}{z^3} \partial_z v(z) \right) + \frac{(3-\gamma_m)(1+\gamma_m)}{z^5} v(z) = 0 \, . \tag{18}$$

Solution for $0 < \gamma_m < 1$ is given by #5

$$v(z)^{(\epsilon)} = c_1 \left(\frac{z}{L} \right)^{1+\gamma_m} + c_2 \left(\frac{z}{L} \right)^{3-\gamma_m} , \tag{19}$$

where c_1 and c_2 are determined by the boundary conditions (8) and (10) as

$$c_1 = \frac{\mathcal{M} - \left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m} \left(\frac{L}{z_m}\right)^{1+\gamma_m} \frac{\xi}{L}}{\left(1 - \left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m}\right)} , \tag{20}$$

$$\begin{aligned} c_2 &= \frac{1}{L} \frac{\left(\frac{L}{z_m}\right)^{3-\gamma_m} \xi - \left(\frac{L}{z_m}\right)^{2-2\gamma_m} L\mathcal{M}}{\left(1 - \left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m}\right)} \\ &= \left(\frac{L}{z_m}\right)^{3-\gamma_m} \frac{\xi}{L} - \left(\frac{L}{z_m}\right)^{2-2\gamma_m} c_1 . \end{aligned} \tag{21}$$

In the continuum limit $\epsilon \to 0$ the solution takes the form

$$v(z) = M \left(\frac{z}{L} \right)^{1+\gamma_m} + \Sigma \left(\frac{z}{L} \right)^{3-\gamma_m} , \tag{22}$$

where $\lim_{\epsilon\to 0} c_1 = \lim_{\epsilon \to 0} \mathcal{M} = M$ and $\lim_{\epsilon \to 0} c_2 = \Sigma$ are quantities renormalized at the scale $1/L$ and we may write $c_1 = M/\left(1 - (\epsilon/z_m)^{2-2\gamma_m}\right)$ and $c_2 = \Sigma$. In terms of the renormalized quantities Eqs.(20) and (21) are rewritten as:

$$M = \mathcal{M} - \left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m} \left(\frac{L}{z_m}\right)^{1+\gamma_m} \frac{\xi}{L} , \tag{23}$$

$$\Sigma = \left(\frac{L}{z_m}\right)^{3-\gamma_m} \frac{\xi}{L} - \frac{\left(\frac{L}{z_m}\right)^{2-2\gamma_m}}{1 - \left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m}} M \, . \tag{24}$$

In the chiral symmetric limit $M = 0$, we have

$$\Sigma = \left(\frac{L}{z_m}\right)^{3-\gamma_m} \frac{\xi}{L} . \tag{25}$$

By substituting Eq.(22) into Eq.(17), the generating functional for $\langle \bar{T}T \rangle$ is expressed as

$$W[\mathcal{M}] = \int d^4x \frac{L}{2g_5^2} \left[\frac{-L^2}{z^3} \partial_z v(z) \cdot v(z) \right]_\epsilon^{z_m} . \tag{26}$$

#5 If we set $\gamma_m \equiv 1$ in Eq.(18), we find a solution $v(z) = C_1 z^2 + C_2 z^2 \ln z$, which was used in analysis in Refs.[27, 28]. Here we understand $\gamma_m = 1$ as the limit $\gamma_m \to 1 - 0$ which implies $C_2 \to 0$ [28], namely, $v(z) = \Sigma \left(\frac{z}{L}\right)^2 = \left(\frac{L}{z_m}\right)^2 \frac{\xi}{L} \left(\frac{z}{L}\right)^2$ as seen from Eq.(25). The other choice $C_1 = 0$ was adopted in Ref.[27]. See Appendix A for discussion on this point.

From Eq.(14) we find the techni-fermion condensate $\langle \bar{T}T \rangle$:

$$\langle \bar{T}T \rangle = -\frac{1}{L^3}\frac{L}{g_5^2}(3-\gamma_m)\left(\frac{L}{z_m}\right)^{3-\gamma_m}\xi\,, \tag{27}$$

where we have used Eq.(21) to rewrite Σ in terms of ξ and M. From this form, we see that the IR value ξ is actually associated with the techni-fermion condensate $\langle \bar{T}T \rangle$ (in a combination with z_m, however). From Eqs.(25) and (27) we see that Σ is more directly related to $\langle \bar{T}T \rangle$ (without combination with z_m) as

$$\Sigma = -\frac{g_5^2/L}{3-\gamma_m}\cdot \langle \bar{T}T \rangle L^2\,. \tag{28}$$

B. Generating Functional $W[v_\mu, a_\mu]$ and $\Pi_{V,A}$

Under the gauge-fixing condition (7), we may find the equations of motion for the transversely polarized component of the gauge fields $V_\mu(x,z)$ and $A_\mu(x,z)$,

$$\left[\partial^2 - z\partial_z \frac{1}{z}\partial_z\right] V_\mu(x,z) = 0\,, \tag{29}$$

$$\left[\partial^2 - z\partial_z \frac{1}{z}\partial_z + \frac{2L^2v^2(z)}{z^2}\right] A_\mu(x,z) = 0\,. \tag{30}$$

In solving these equations, it is convenient to perform partially Fourier transformation on $V_\mu(x,z)$ and $A_\mu(x,z)$ with respect to x_μ,

$$V_\mu(x,z) = \int_q e^{iqx} V_\mu(q,z)\,, \qquad A_\mu(x,z) = \int_q e^{iqx} A_\mu(q,z)\,, \tag{31}$$

where the Fourier components $V_\mu(q,z)$ and $A_\mu(q,z)$ may be decomposed as

$$V_\mu(q,z) = \tilde{v}^\mu(q)\, V(q,z)\,, \qquad A_\mu(q,z) = \tilde{a}^\mu(q)\, A(q,z)\,. \tag{32}$$

Putting these into Eqs.(29) and (30), we have

$$\left[q^2 + z\partial_z \frac{1}{z}\partial_z\right] V(q,z) \;=\; 0, \tag{33}$$

$$\left[q^2 + z\partial_z \frac{1}{z}\partial_z - \frac{2L^2v^2(z)}{z^2}\right] A(q,z) \;=\; 0, \tag{34}$$

with the boundary condition

$$\partial_z V(q,z_m) \;=\; \partial_z A(q,z_m) \;=\; 0, \tag{35}$$

$$V(q,\epsilon) \;=\; A(q,\epsilon) \;=\; 1. \tag{36}$$

The generating functional $W[v_\mu, a_\mu]$ is now expressed in terms of $V(q,z)$ and $A(q,z)$ as follows:

$$W[v_\mu, a_\mu] = \frac{1}{2}\int_q \frac{-L}{g_5^2\epsilon}\mathrm{Tr}\left[\tilde{v}_\mu(-q)\partial_z V(q,\epsilon)\cdot\tilde{v}^\mu(q) + \tilde{a}_\mu(-q)\partial_z A(q,\epsilon)\cdot\tilde{a}^\mu(q)\right]\,. \tag{37}$$

Accordingly, the vector and axialvector current correlators Π_V and Π_A, defined as in Eqs.(12) and (13), take the form:

$$\Pi_V(Q^2) = \frac{L}{g_5^2\epsilon}\partial_z V(Q^2,\epsilon)\,, \qquad \Pi_A(Q^2) = \frac{L}{g_5^2\epsilon}\partial_z A(Q^2,\epsilon)\,, \tag{38}$$

where we have rewritten $V(q,z) = V(Q^2,z)$ and $A(q,z) = A(Q^2,z)$.

It is now obvious that the chiral symmetry breaking effects described by $\Pi_V(Q^2) - \Pi_A(Q^2)$ are related to $\partial_z V(Q^2,\epsilon) - \partial_z A(Q^2,\epsilon)$ and hence arise only from the $v(z)$ term in Eq.(30) which is the unique origin of the γ_m-dependence in this approach. If the chiral symmetry gets restored $\langle \bar{T}T \rangle \to 0$ such that $v(z) \to 0$, we should get $[\Pi_V(Q^2) - \Pi_A(Q^2)] \to 0$,

and hence at first glance, its derivative $\hat{S} \sim \frac{\partial}{\partial Q^2}\left[\Pi_V(Q^2) - \Pi_A(Q^2)\right]\Big|_{Q=0}$ would also vanish. However, overall absolute value of $\Pi_V(Q^2) - \Pi_A(Q^2)$ is normalized by $f_\pi^2 = [\Pi_V(0) - \Pi_A(0)] \to 0$ so that $[\Pi_V(Q^2) - \Pi_A(Q^2)] \to 0$ is realized even with $\hat{S} \neq 0$. Of course, if we have $[\Pi_V(Q^2) - \Pi_A(Q^2)] \equiv 0$, then $\hat{S} \equiv 0$ as it should. The situation is very much like the perturbative calculation of $\hat{S}$: There could be discontinuity at the phase transition point. It should also be noted that although $v(z) \to 0$ in the chiral limit, $M = 0$, implies $\Sigma \to 0$ and $\langle \bar{T}T \rangle \to 0$, it does not necessarily $\xi \to 0$. This is because that $1/z_m \to 0$ is also possible in the expression of Eq.(27). The $\hat{S}$ is given as a function of ξ which is a combination of $\langle \bar{T}T \rangle$ and z_m, so that its behavior near the conformal phase transition is not directly connected with $\langle \bar{T}T \rangle \to 0$. In the following sections we shall discuss these points carefully.

1. *Vector Current Correlator* Π_V

A solution of Eq.(33) with the boundary conditions (35) and (36) taken into account is given by the modified Bessel functions I and K,

$$V(Q^2, z) = \frac{K_0(Qz_m) \cdot I_1(Qz) + I_0(Qz_m) \cdot K_1(Qz)}{I_0(Qz_m) \cdot K_1(Q\epsilon) + K_0(Qz_m) \cdot I_1(Q\epsilon)}\,, \tag{39}$$

and hence Π_V in Eq.(38) is given by

$$\Pi_V(Q^2) = \frac{L}{g_5^2}\frac{Q}{\epsilon}\frac{K_0(Qz_m) \cdot I_0(Q\epsilon) - I_0(Qz_m) \cdot K_0(Q\epsilon)}{I_0(Qz_m) \cdot K_1(Q\epsilon) + K_0(Qz_m) \cdot I_1(Q\epsilon)}\,. \tag{40}$$

In particular, for small Q^2, we may calculate approximately

$$\Pi_V(Q^2 \to 0) \sim -\frac{LQ^2}{g_5^2}\log\left(\frac{z_m}{\epsilon}\right). \tag{41}$$

We will later come back to this expression in evaluating the S parameter.

2. *Axialvector Current Correlator* Π_A

To derive the solution for Π_A in an analytic manner [27], we may define the following quantity:

$$P(Q^2, z) = \frac{1}{z}\partial_z \log A(Q^2, z)\,, \tag{42}$$

in terms of which Π_A is expressed as

$$\Pi_A(Q^2) \;=\; \frac{L}{g_5^2}P(Q^2, \epsilon). \tag{43}$$

Equation of motion (34) is rewritten as

$$z\partial_z P(Q^2, z) + z^2 P(Q^2, z)^2 - Q^2 - 2\left(\frac{L\,v(z)}{z}\right)^2 = 0, \tag{44}$$

with the boundary condition

$$P(Q^2, z_m) = 0. \tag{45}$$

We expand $P(Q^2, z)$ perturbatively in powers of Q^2 as

$$P(Q^2, z) = P(0, z) + Q^2 P'(0, z) + \cdots\,, \tag{46}$$

where $P'(0, z) \equiv \partial P(Q^2, z)/\partial Q^2\Big|_{Q^2=0}$. These expansion coefficients are determined by solving the following equations order by order in Q^2:

$$O(Q^0) \;:\; z\partial_z P(0, z) + z^2\left(P(0, z)\right)^2 = \frac{2L^2 v^2(z)}{z^2}\,, \tag{47}$$

$$O(Q^2) \;:\; z\partial_z P'(0, z) + 2z^2 P(0, z)P'(0, z) = 1\,, \tag{48}$$

which are derived from Eqs.(44) and (46). Inserting into Eq.(47) the solution of $v(z)$ given in Eq.(22) with $M = 0$ taken, we may find a solution of Eq.(47) so as to satisfy the boundary condition (45):

$$P(0,z) = \frac{\Delta X(z)}{z\epsilon} \frac{I_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot K_{\frac{1-\Delta}{\Delta}}(X(z)) - K_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot I_{\frac{1-\Delta}{\Delta}}(X(z))}{I_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot K_{\frac{1}{\Delta}}(X(\epsilon)) + K_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot I_{\frac{1}{\Delta}}(X(\epsilon))} \,, \tag{49}$$

with $\Delta = 3 - \gamma_m$ and $X(z) = \frac{\sqrt{2}\xi}{3-\gamma_m}\left(\frac{z}{z_m}\right)^{3-\gamma_m}$. Using that solution, we successively solve Eq.(48):

$$P'(0,z) = \int_z^{z_m} \frac{d\,z'}{z'} \left(A(0,z')\right)^2 \,, \tag{50}$$

where

$$A(0,z) = \frac{z}{\epsilon} \frac{I_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot K_{\frac{1}{\Delta}}(X(z)) + K_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot I_{\frac{1}{\Delta}}(X(z))}{I_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot K_{\frac{1}{\Delta}}(X(\epsilon)) + K_{\frac{1-\Delta}{\Delta}}(X(z_m)) \cdot I_{\frac{1}{\Delta}}(X(\epsilon))} \,. \tag{51}$$

Then $\Pi_A(0)$ and $\Pi'_A(0)$ are respectively given by the expansion coefficients $P(0,z)$ and $P'(0,z)$:

$$\begin{aligned}
\Pi_A(0) &= \frac{L\,P(0,\epsilon)}{g_5^2} \\
&= \frac{L}{g_5^2}\frac{2\Delta}{z_m^2}\left(\frac{\xi}{\sqrt{2}\Delta}\right)^{2/\Delta} \frac{\Gamma(\frac{\Delta-1}{\Delta})}{\Gamma(\frac{1}{\Delta})}\left(\frac{2}{\pi}\sin\left(\frac{\pi}{\Delta}\right)\frac{K_{\frac{1-\Delta}{\Delta}}(X(z_m))}{I_{\frac{1-\Delta}{\Delta}}(X(z_m))} - 1\right) + \mathcal{O}\left(\left(\frac{\epsilon}{z_m}\right)^{2\Delta}\right), && (52) \\
\Pi'_A(0) &= \frac{L\,P'(0,\epsilon)}{g_5^2} = \frac{L}{g_5^2}\int_\epsilon^{z_m}\frac{d\,z'}{z'}\left(A(0,z')\right)^2 \,. && (53)
\end{aligned}$$

$\Pi_A(0)$ yields the decay constant f_π, $\Pi_A(0) = -f_\pi^2$, while $\Pi'_A(0)$ is related to $\hat{S}$: $\hat{S} = -4\pi\left[\Pi'_V(0) - \Pi'_A(0)\right]$.

III. THE S PARAMETER IN HOLOGRAPHIC TECHNICOLOR

Now that we have calculated two-point functions Π_V and Π_A, we can compute $\hat{S}$ defined as in Eq.(16):

$$\begin{aligned}
\hat{S} &= \frac{4\pi L}{g_5^2}\left[\log\frac{z_m}{\epsilon} - z\int_\epsilon^{z_m}\frac{dz'}{z'}\left(A^{(0)}(z')\right)^2\right] \\
&= \frac{4\pi L}{g_5^2}\int_\epsilon^{z_m}\frac{dz'}{z'}\left[1 - \left(A^{(0)}(z')\right)^2\right] \,, && (54)
\end{aligned}$$

where use has been made of Eqs.(41) and (53). We now evaluate $\hat{S}$ and show that $\hat{S}$ depends only on the ratio f_π/M_ρ in the limit $\epsilon \to 0$, once the value of γ_m is fixed, where M_ρ and f_π denote respectively the mass of techni ρ meson and the decay constant.

A. Parameters Relevant to $\hat{S}$

Let us first recall that the original 5-dimensional holographic model analyzed in this paper is described by six parameters, (L/g_5^2), z_m, ϵ, γ_m, ξ, and $\mathcal{M}$ (or M). As far as calculation of the S parameter is concerned, it is sufficient to work in the chiral limit $M = 0$, in which case $\mathcal{M}$ is related to ξ as seen in Eq.(23). It should also be noted that the dimensionful parameters ϵ and z_m enter the dimensionless quantity $\hat{S}$ only through the ratio ϵ/z_m. In the TC scenario ϵ is taken to be the ETC scale $1/\epsilon = \Lambda_{\rm ETC}$ and hence $\epsilon/z_m \ll 1$. Here we simply put $\epsilon/z_m = 0$. On the other hand, the decay constant f_π, $f_\pi^2 = -\Pi_A(0)$, depends solely on the dimensionful parameter z_m.

The number of parameters relevant to $\hat{S}$ and f_π^2 hence results in three and four, respectively:

$$\begin{aligned}
\hat{S} &= \hat{S}\left(L/g_5^2;\, \gamma_m;\, \xi\right) \,, && (55) \\
f_\pi^2 &= f_\pi^2\left(L/g_5^2;\, \gamma_m;\, \xi;\, z_m\right) \,. && (56)
\end{aligned}$$

Although the holography gives us $\hat{S}$ and f_π^2 as functions of these parameters as in Eqs.(40), (52), and (53), values of the parameters are not calculable in the framework of the present holographic approach. In the following, thereby, we shall discuss how these parameters would behave in the framework of walking/conformal TC with large anomalous dimension γ_m.

1. Parameter L/g_5^2

The parameter (L/g_5^2) can be determined by requiring that the high energy behavior of the current correlator Π_V should match with those of corresponding TC theory in large N_{TC} limit. Let us look at the large momentum behavior of Π_V derived from Eq.(40):

$$\Pi_V(Q^2 \to \infty) \sim \frac{LQ^2}{2g_5^2} \log{(Q^2\epsilon^2)} \,. \tag{57}$$

This expression may be matched with that calculated from the operator product expansion (OPE) in the large N_{TC} limit,

$$\Pi_V(Q^2 \to \infty) \sim \frac{N_{TC}Q^2}{24\pi^2} \log{(Q^2\epsilon^2)} \,, \tag{58}$$

so that (L/g_5^2) is determined as [20, 21]

$$\frac{L}{g_5^2} = \frac{N_{TC}}{12\pi^2} \,. \tag{59}$$

Thus it turns out that once N_{TC} is fixed, $\hat{S}$ and f_π^2 depend only on two and three parameters, respectively:

$$\begin{aligned} \hat{S} &= \hat{S}(\gamma_m;\xi) \\ &= \frac{N_{TC}}{3\pi} \int_\epsilon^{z_m} \frac{dz'}{z'} \left[1 - \left(A^{(0)}(z') \right)^2 \right] \,, \end{aligned} \tag{60}$$

$$f_\pi^2 = f_\pi^2\,(\gamma_m;\, \xi;\, z_m) \,, \tag{61}$$

where we have used Eq.(59). Notice from Eq.(51) that the ξ- and γ_m-dependences are embedded in the expression of $A^{(0)}$.

2. Parameters ξ and z_m

From Eq.(61) we see that the parameter ξ may be related to f_π together with the IR brane position z_m. Equation (52) takes the form for $\epsilon/z_m \to 0$

$$f_\pi^2 = \frac{L}{g_5^2} \frac{2\Delta}{z_m^2} \left(\frac{\xi}{\sqrt{2}\Delta} \right)^{2/\Delta} \frac{\Gamma(\frac{\Delta-1}{\Delta})}{\Gamma(\frac{1}{\Delta})} \left(1 - \frac{2}{\pi} \sin\left(\frac{\pi}{\Delta}\right) \frac{K_{\frac{1-\Delta}{\Delta}}(X(z_m))}{I_{\frac{1-\Delta}{\Delta}}(X(z_m))} \right) \,, \tag{62}$$

with $\Delta \equiv 3 - \gamma_m$ and $X(z_m) = \sqrt{2}\xi/(3-\gamma_m)$. See Figures. 1 and 2. Note that f_π is a monotonically increasing function of ξ and γ_m.

For $\xi \gg 1$ Eq.(62) takes the form [20]:

$$f_\pi^2 \overset{\xi \gg 1}{\simeq} \frac{L}{g_5^2} 2^{(1-1/\Delta)} \Delta^{(1-2/\Delta)} \frac{\Gamma(1-1/\Delta)}{\Gamma(1/\Delta)} \frac{\xi^{2/\Delta}}{z_m^2} \,. \tag{63}$$

On the other hand, for $\xi \ll 1$ we have

$$f_\pi^2 \overset{\xi \ll 1}{\simeq} \frac{L}{g_5^2} \frac{1}{\Delta - 1} \frac{\xi^2}{z_m^2} \,, \tag{64}$$

which coincides with Ref. [28] in the case of $\gamma_m = 1$ under certain condition.

The parameter z_m may be related to a typical vector meson mass scale in the walking/conformal TC. To see this explicitly, we expand Π_V in terms of the vector meson poles together with the pole resides,

$$\Pi_V(Q^2) = -Q^2 \sum_n \frac{F_{V_n}^2}{Q^2 + M_{V_n}^2} \,, \tag{65}$$

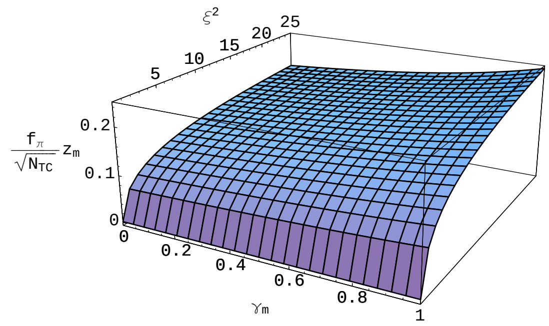

FIG. 1: 3-dimensional plot of $\frac{f_\pi}{\sqrt{N_{TC}}} z_m$ drawn on (γ_m, ξ^2)-plane.

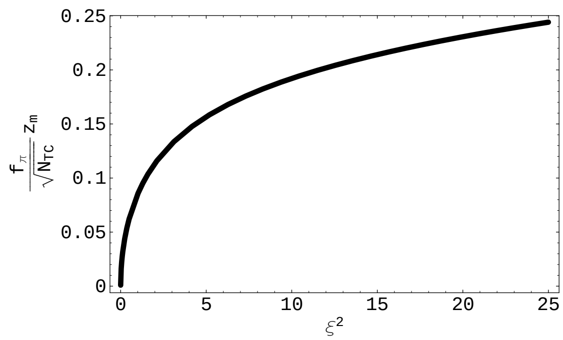

FIG. 2: Plot of ξ^2-dependence of $\frac{f_\pi}{\sqrt{N_{TC}}} z_m$ with $\gamma_m \simeq 1$.

where F_{V_n} and M_{V_n} denote respectively the vector meson decay constants and their masses. Extracting the lightest vector meson-pole, i.e., techniρ-pole, in Eq.(40) and comparing that with Eq.(65), we find [20, 21]

$$M_{V_1} \equiv M_\rho \simeq \frac{2.4}{z_m} \,. \tag{66}$$

Using Eqs.(63), (64), and (66), we have

$$\xi^2 \simeq \left(\frac{(2.4)^2 C(\gamma_m)}{N_{TC}} \cdot \frac{f_\pi^2}{M_\rho^2} \right)^{3-\gamma_m} , \qquad \text{for } \xi \gg 1 \,, \tag{67}$$

$$\xi^2 \simeq \frac{12\pi^2 (2.4)^2 (2-\gamma_m)}{N_{TC}} \cdot \frac{f_\pi^2}{M_\rho^2} \,, \qquad \text{for } \xi \ll 1 \,, \tag{68}$$

where C is a numerical factor depending on γ_m $(= 3 - \Delta)$,

$$C(\gamma_m) = 12\pi^2 2^{(1/\Delta - 1)} \Delta^{(2/\Delta - 1)} \frac{\Gamma(1/\Delta)}{\Gamma(1-1/\Delta)} . \tag{69}$$

B. $\hat{S}$ from Holographic Calculation

We are now ready to evaluate $\hat{S}$ written as a function of ξ with large anomalous dimension γ_m not restricted to $\gamma_m = 1$. Using Eqs.(54) and (59), we numerically compute $\hat{S}$, the result given in Figs. 3 and 4. This is our main result.

Varying the values of ξ and γ_m, in Fig. 3 we draw a 3-dimensional plot of $\hat{S}/N_{\rm TC}$ as a function of ξ and γ_m. Also, a plot on ($\hat{S}/N_{\rm TC}$, ξ^2)-plane for $\gamma_m \simeq 1$ is shown in Fig. 4. Looking at Figs. 3 and 4, we find that for smaller value

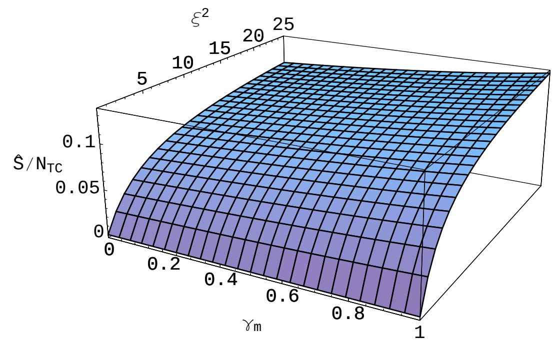

FIG. 3: 3-dimensional plot of $\hat{S}/N_{TC}$ drawn on (γ_m, ξ^2)-plane.

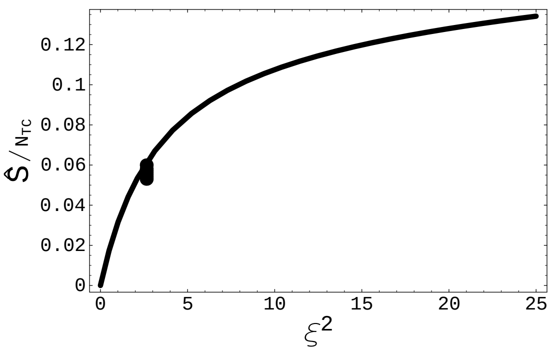

FIG. 4: Plot of ξ^2-dependence of $\hat{S}/N_{TC}$ with $\gamma_m \simeq 1$. The blob is the result of the ladder SD and BS equations, ξ from homogeneous BS equation [17] and $\hat{S}$ from inhomogeneous BS equation [4] (to be explained in Sec.V A 1).

of $\xi(\ll 1)$ $\hat{S}$ slightly decreases as γ_m increases, while for larger value of $\xi(\gg 1)$ it slightly increases as γ_m increases. Thus there is no dramatic dependence on γ_m of $\hat{S}$ as a function of ξ or f_π/M_ρ, although γ_m dependence could enter in ξ itself (we later establish that this is not the case at least explicitly).

In the case of QCD with $\gamma_m \simeq 0$, by using phenomenological inputs $f_\pi \simeq 92.4$ MeV and $M_\rho \simeq 775$ MeV, we estimate $\xi^2 \simeq (4.82)^2$ by Eqs.(59), (62), and (66), which in Fig. 3 implies $\hat{S} \simeq 0.30$ in agreement with the experiment $\hat{S} \simeq 0.32$. So the QCD monitor of this holographic approach is checked. The result is consistent with the estimate of Ref. [20, 27].

Most remarkably, we find from Figs. 3 and 4 that $\hat{S}$ decreases monotonically with respect to ξ, i.e., f_π/M_ρ, namely $\hat{S} \to 0$ can be achieved by taking $\xi \to 0$, or equivalently, $f_\pi/M_\rho \to 0$. This tendency is in accord with Ref. [27, 28] with $\gamma_m = 1$ and also with our later discussion. This implies that the holography provides a novel avenue to having a small S. Then the next issue is whether or not we can realize ξ in the W/C TC with $\gamma_m \simeq 1$ much smaller than $\xi \simeq 4.82$ of QCD with $\gamma_m \simeq 0$ [#6].

Actually, the experimental constraint on $\hat{S}$ for W/C TC is $S = \frac{N_f}{2} \cdot \hat{S} < 0.1$. In the case of typical W/C TC with $N_{TC} = 2$ and $N_f = 8$ we need $\hat{S} < 0.025$ and hence [#7]

$$\xi^2 < (0.59)^2\,, \ \text{or} \ \frac{f_\pi}{M_\rho} < 0.038 \tag{70}$$

[#6] Our result appears to imply that, even with small anomalous dimension $\gamma_m \simeq 0$ as in QCD, $\hat{S}$ could be vanishingly small if f_π/M_ρ were arranged to vanish. The point is that in QCD-like theories there is actually no chiral phase transition where f_π/M_ρ could vanish. In contrast, walking/conformal TC characterized by the BZ-IR fixed point does have chiral phase transition where f_π/M_ρ could have chance to vanish at the phase transition point. This point will be discussed in details in later section.

[#7] In the case of "minimal walking" [15], the constraint could be somewhat weaker.

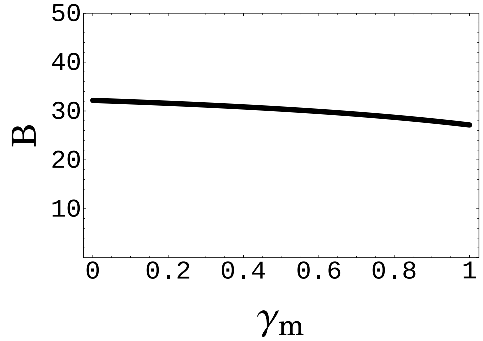

FIG. 5: Plot of the γ_m-dependence of B (see text).

from Fig. 4, which corresponds to the techni-ρ mass $M_\rho > 3.3$TeV. In the later section we shall discuss whether or not the situation $\xi \ll 1$ can be realized in W/C TC with $\gamma_m \simeq 1$.

For later convenience let us next derive an analytic expression of $\hat{S}$ for $\xi \gg 1$ and $\xi \ll 1$. For $\xi \gg 1$, from Eq.(54) with Eq.(51), we have

$$\hat{S} \quad \stackrel{\xi \gg 1}{\simeq} \quad \frac{N_{TC}}{3\pi(3-\gamma_m)} \ln \xi \,, \tag{71}$$

which is in accord with the expression obtained in Ref.[20]. As read off from Eq.(71), $\hat{S}$ cannot be smaller than $\mathcal{O}(1)$ in the case of $\xi \gg 1$, which indicates that this case is phenomenologically unacceptable for TC.

We turn to the case that $\xi \ll 1$:

$$\hat{S} \quad \stackrel{\xi \ll 1}{\simeq} \quad \frac{N_{TC}}{6\pi} \frac{4-\gamma_m}{(3-\gamma_m)^2} \xi^2 \,. \tag{72}$$

It is interesting to note that the right hand side of Eq.(72) is rewritten by using Eq.(68):

$$\begin{aligned} \hat{S} &\simeq B \cdot \left(\frac{f_\pi}{M_\rho} \right)^2 , \qquad \text{for } f_\pi/M_\rho \ll 1, \\ B &= 2\pi (2.4)^2 \frac{(2-\gamma_m)(4-\gamma_m)}{(3-\gamma_m)^2}, \end{aligned} \tag{73}$$

which shows that $\hat{S}$ is given as a function of f_π/M_ρ once γ_m is specified. As already noted in the numerical calculation of $\hat{S}$, B in the analytical form is fairly independent of γ_m unless ξ or f_π/M_ρ is subject to further substantial reduction due to large γ_m (we shall discuss this point later): $B \simeq 27$ for $\gamma_m \simeq 1$ while $B \simeq 32$ as the QCD monitor value for $\gamma_m \simeq 0$ (see Fig. 5).

The authors in Ref. [27] numerically computed $\hat{S}$ as a function of f_π/M_ρ, focusing only on the case with $\gamma_m = 1$ (besides $\gamma_m = 0$) #8. The analytic calculation of $\hat{S}$ was also done in Ref. [28] with $\gamma_m = 1$, which resulted in the form $\hat{S} \sim (f_\pi/M_\rho)^2$ for $f_\pi/M_\rho \ll 1$. Equation (73) is the analytical expression of our main result. It implies that $\hat{S} \to 0$ as $f_\pi/M_\rho \to 0$ fairly independently of γ_m. This suggests existence of a class of phenomenologically viable models of walking/conformal TC with $\gamma_m \simeq 1$.

At this point one might suspect that our result (73) is rather trivial, since we already know (see e.g. Ref. [25]) the ρ-pole-dominated expression for $\hat{S}$ (or $L_{10} = -\hat{S}/(16\pi)$):

$$\hat{S} \simeq 4\pi \frac{F_\rho^2}{M_\rho^2} = 4\pi a \frac{f_\pi^2}{M_\rho^2} \, \left(= \left(\frac{g^2}{4\pi} \right)^{-1} \right) , \tag{74}$$

#8 The numerical result of Ref.[27] with $C_1 = 0$ (see footnote #5) is somewhat different from ours which corresponds to $C_2 \to 0$ in the limit $\gamma_m \to 1$. Subtlety about $\gamma_m = 1$ will be discussed in Appendix A

where $F_\rho\,(=\sqrt{a}f_\pi)$ is the decay constant of the ρ meson (or, that of the NG boson absorbed into longitudinal ρ in the HLS model language), with $a \simeq 2$ in QCD case by the experiments, and g the gauge coupling constant of the HLS. This actually scales as $\hat{S} \sim f_\pi^2/M_\rho^2$. What is nontrivial with our result in this paper is that the holographic calculation includes all the contributions of the poles not restricted to the lowest one and yet the coefficient B has no nontrivial dependences on other parameters. To see this we may write $\hat{S}$ in terms of the vector and the axialvector meson pole-dominated expression,

$$\hat{S} = 4\pi \sum_{n=1}^{\infty} \left(\frac{A_{V_n}}{c_{V_n}} - \frac{A_{A_n}}{c_{A_n}} \right) \cdot \left(\frac{f_\pi}{M_\rho} \right)^2 , \tag{75}$$

where

$$A_{V_n,A_n} \equiv \left(\frac{F_{V_n,A_n}}{f_\pi} \right)^2 , \qquad c_{V_n,A_n} \equiv \left(\frac{M_{V_n,A_n}}{M_\rho} \right)^2 , \tag{76}$$

with F_{V_n,A_n} and M_{V_n,A_n} being respectively the (axial-)vector meson decay constants and (axial-)vector meson masses. Note that, although each coefficient A_{V_n,A_n} and c_{V_n,A_n} cannot easily be calculated without solving non-perturbative issues, we may compute a sum of infinite set of pole contributions comparing Eq.(73) with Eq.(75):

$$\sum_{n=1}^{\infty} \left(\frac{A_{V_n}}{c_{V_n}} - \frac{A_{A_n}}{c_{A_n}} \right) = \frac{B}{4\pi} . \tag{77}$$

IV. ESTIMATION OF ξ, OR f_π/M_ρ

In the previous section, we showed that in the holographic calculation $\hat{S}$ is a monotonically increasing function of ξ and $\hat{S} \to 0$ as $\xi \to 0$ (Eq.(72)), fairly independently of γ_m. Thus the problem is whether or not the situation $\xi \ll 1$ as in Eq.(70) can be realized in the holographic W/C TC. We may recall that ξ is related to the techni-fermion condensate $\langle \bar{T}T \rangle$ as in Eq.(27) and also to f_π/M_ρ as in Eq.(62) or Eqs.(67) and (68). In the following, through correct identification of renormalization-point of $\langle \bar{T}T \rangle$, we shall demonstrate that ξ has no particular suppression factor due solely to γ_m and so does S in contrast to previous authors [27, 28]. Hence the situation $\xi \ll 1$ can only be realized near the conformal phase transition point i.e., chiral symmetry restoration point at the conformal window where $\gamma_m \to 1$ and vanishing of the dynamical mass m of techni-fermion, $m \to 0$, may be correlated. Actually the straightforward dynamical calculation based on the ladder SD and BS equations [4] shows that this does happen in contrast to the holographic calculation performed here.

A. ξ and Renormalization of $\langle \bar{T}T \rangle$

1. *Renormalization of $\langle \bar{T}T \rangle$*

In order to see whether or not a nontrivial dependence of f_π/M_ρ on γ_m exists without referring to the conformal phase transition $m \to 0$, we shall make a correct identification of the renormalization-point of $\langle \bar{T}T \rangle$. Let us go back to the expression of the current mass of techni-fermion M and ξ given in Eqs.(8) and (10). In the case $\gamma_m < 1$ #9 we can safely neglect the second term of Eq.(23) for $\epsilon \to 0$:

$$M = \mathcal{M} = \left(\frac{1/L}{1/\epsilon} \right)^{-\gamma_m} \cdot \left(\frac{L}{\epsilon} v(\epsilon) \right) . \tag{78}$$

Given the anomalous dimension $\gamma_m \equiv \partial \ln Z_m(\mu)/\partial \ln \mu$, we obtain the mass renormalization constant $Z_m = \left(\frac{1/L}{1/\epsilon} \right)^{\gamma_m}$ by integration from the cutoff scale $1/\epsilon$ down to the infrared scale $1/L$ in a standard way. Then from Eq. (78) we see that M is the current mass renormalized at $1/L$ and

$$M_0 \equiv Z_m\, M = \left(\frac{1/L}{1/\epsilon} \right)^{\gamma_m} M = \frac{L}{\epsilon} v(\epsilon) \tag{79}$$

#9 We shall discuss the limit of $\gamma_m \to 1$ in the Appendix

is the bare mass at the cutoff scale. We may also introduce a bare condensate:

$$\langle \bar{T}T \rangle_0 = i\frac{i\delta W[M_0]}{\delta M_0}\bigg|_{M_0=0} = \left(\frac{1/L}{1/\epsilon}\right)^{-\gamma_m} \cdot \langle \bar{T}T \rangle = Z_m^{-1} \langle \bar{T}T \rangle \,, \tag{80}$$

where $\langle \bar{T}T \rangle$ is given by Eq.(27). Then we have a standard multiplicative renormalization $M_0 \langle \bar{T}T \rangle_0 = M \langle \bar{T}T \rangle$. Since $M \equiv M_{1/L}$ is the external source for $\langle \bar{T}T \rangle$, we conclude that $\langle \bar{T}T \rangle$ in Eq.(27) is nothing but the techni-fermion condensate renormalized at the $1/L$, $\langle \bar{T}T \rangle \equiv \langle \bar{T}T \rangle_{1/L}$.

Hence the expression of ξ can be written by solving Eq.(27) inversely as

$$\xi \;=\; -\frac{12\pi^2}{N_{TC}} \frac{z_m^3}{3-\gamma_m} \left(\frac{L}{z_m}\right)^{\gamma_m} \langle \bar{T}T \rangle_{1/L} \,, \tag{81}$$

where we have used the matching condition of Eq.(59). We may define the techni-fermion condensate renormalized at $1/z_m$, $\langle \bar{T}T \rangle_{1/z_m} \equiv \left(\frac{L}{z_m}\right)^{\gamma_m} \langle \bar{T}T \rangle_{1/L}$, in terms of which we rewrite ξ as

$$\xi = -\frac{12\pi^2}{N_{TC}} \frac{z_m^3}{3-\gamma_m} \langle \bar{T}T \rangle_{1/z_m} \,, \tag{82}$$

from which we readily see that ξ is independent of the renormalization scale L, and accordingly so is $\hat{S}$ as it should be. This is in sharp contrast to the result of the previous authors [27, 28] which explicitly depends on L: $\xi \sim (L/z_m)$ for $\gamma_m = 1$, with implicit identification of $\langle \bar{T}T \rangle_{1/L}$ as $\langle \bar{T}T \rangle_{1/z_m}$. Thus even if we take $L \to \epsilon$, there is no suppression factor due to γ_m.

2. *Relationship Between f_π, m and M_ρ*

The renormalization-point dependence of $\langle \bar{T}T \rangle$ is further given by [9]

$$\begin{aligned} \langle \bar{T}T \rangle_{1/L} \;&=\; \left(\frac{1/L}{m}\right)^{\gamma_m} \langle \bar{T}T \rangle_m \,, \\ \langle \bar{T}T \rangle_m \;&=\; -\frac{N_{TC}}{4\pi^2} \cdot m^3, \end{aligned} \tag{83}$$

where the dynamical mass of techni-fermion m may be defined as: $m \equiv \Sigma(p^2 = -m^2)$ for $iS_F^{-1}(p) = \not{p} - \Sigma(p^2)$. Note that $m \to 0$ at the conformal phase transition point. Combining Eqs.(81) and (83), we find

$$\xi = \frac{3}{3-\gamma_m} \left(\frac{m}{z_m^{-1}}\right)^{3-\gamma_m} \,. \tag{84}$$

Recalling the relationship between z_m and M_ρ given in Eq.(66), we may further rewrite the right hand side of Eq.(84) as

$$\xi \simeq \frac{3 \cdot (2.4)^{3-\gamma_m}}{3-\gamma_m} \left(\frac{m}{M_\rho}\right)^{3-\gamma_m} \,. \tag{85}$$

Without knowing further information that $m/M_\rho \to 0$ as $m \to 0$ (and $\gamma_m \to 1$), we see from Eq.(85) that ξ has no suppression factor due to γ_m (it even enhances as γ_m increases !).

Incidentally, Eq.(25) is rewritten through Eq.(84) as

$$\Sigma = \left(\frac{L}{z_m}\right)^{3-\gamma_m} \frac{\xi}{L} = \frac{3}{3-\gamma_m} \frac{1}{L} (mL)^{3-\gamma_m} \quad \stackrel{m\to 0}{\longrightarrow} \quad 0 \,, \tag{86}$$

which implies $v(z) = \Sigma (z/L)^{3-\gamma_m} \to 0$ as $m \to 0$, as it should. This should be contrasted with ξ which does not necessarily have a direct tie with the chiral symmetry restoration $m \to 0$.

Equating right-hand sides of Eq.(85) and Eqs.(67)-(68), we find a relation between f_π and m which reads

$$\xi \gg 1 \;:\; f_\pi^2/N_{\rm TC} \sim m^2 \,, \tag{87}$$

$$\xi \ll 1 \;:\; f_\pi^2/N_{\rm TC} \sim m^2 \cdot \left(\frac{m}{M_\rho}\right)^{4-2\gamma_m} \,, \tag{88}$$

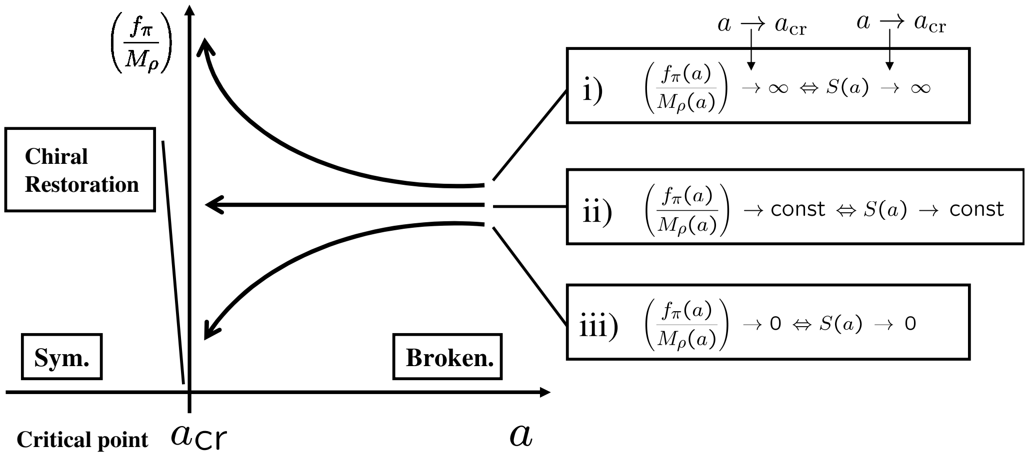

FIG. 6: An illustration of a classification of holographic walking/conformal technicolor models in terms of scaling behaviors of f_π/M_ρ near the edge of conformal window, which represents $a = a_{cr}$ in this figure, where a denotes a tuning parameter characterizing the chiral/conformal phase transition by $a = a_{cr}$. "Broken." and "Sym."respectively stands for the broken and the symmetric phases.

near the conformal phase transition point $m \to 0$. It is worth comparing these relationships with those obtained from the Pagels-Stokar formula [33](For details, see Appendix B).

Equation (88) is a novel scaling when $m/M_\rho \to 0$, namely f_π vanishes much faster than m. As we shall discuss in the following section, the holography as it stands does allow for this possibility, although we know no explicit calculation to have $m/M_\rho \to 0$ at the conformal phase transition.

V. HOLOGRAPHY VERSUS W/C TC

In the previous section, we showed that in the holographic calculation ξ, or f_π/M_ρ has no suppression factor due to γ_m, based on the correct identification of the renormalization-point of $\langle TT \rangle$. Actually, the holographic framework as it stands cannot calculate the ratio f_π/M_ρ which is left arbitrary, while it should be a calculable quantity in principle e.g. in the lattice gauge theory. In this section, comparing our result with explicit computation of f_π/M_ρ in other approaches, we shall argue whether or not $f_\pi/M_\rho \ll 1$ can be realized near the conformal window, namely $f_\pi/M_\rho \to 0$ for $m \to 0$ in W/C TC characterized by the Banks-Zaks infrared fixed point.

Without knowledge of detailed dynamics, we may classify the following possibilities as illustrated in Fig. 6 on how f_π/M_ρ behaves near the conformal phase transition point:

i) $M_\rho \searrow 0$ *faster than* $f_\pi \searrow 0$ ($f_\pi/M_\rho \to \infty$). In this case $\hat{S}$ grows to diverge when $m \to 0$.

ii) $M_\rho \searrow 0$ *as fast as* $f_\pi \searrow 0$ ($f_\pi/M_\rho \to$constant). In this case $\hat{S} \to$ constant $\neq 0$, even when $m \to 0$.

iii) $M_\rho \searrow 0$ *slower* than $f_\pi \searrow 0$ ($f_\pi/M_\rho \to 0$). In this case $\hat{S}$ decreases resulting in $\hat{S} = 0$ when $m \to 0$.

From Eqs.(87) and (88) we can read the scaling behavior of f_π versus m near the conformal phase transition point $m \to 0$ in each case. In the cases i) and ii) we have a usual scaling $f_\pi/m \to$ constant, while in the case iii) where $f_\pi/M_\rho \to 0$ we find a novel scaling behavior of f_π, $f_\pi/m \to 0$.

A. Searching For Explicit Dynamics

Keeping in our mind the scaling laws of f_π, given in Eqs.(87) and (88), in the following we will discuss how the conformal phase transition, classified into three types as illustrated in Fig. 6, can be realized in explicit W/C TC dynamics.

1. Example for Case i)

The conformal phase transition corresponding to the case i) may be realized by Vector Manifestation (VM) [25, 30] in the HLS model. The S parameter in HLS model is given by Eq.(74). In order that $\hat{S} < 0.1$ (more realistically $\hat{S} < 0.025$), we would naively need very strong HLS coupling $\frac{g^2}{4\pi} > 10$ (or $\frac{g^2}{4\pi} > 40$!), independently of the tuning of the parameter a. This is quite opposite to what is realized in VM where we have $g \sim \langle \bar{T}T \rangle_\Lambda / \Lambda^3 \sim (m/\Lambda)^{3-\gamma_m} = (m/\Lambda)^2 \to 0$ as we approach the conformal phase transition point, $m \to 0$. Hence we have

$$\hat{S} = 4\pi a \left(\frac{f_\pi}{M_\rho} \right)^2 = \left(\frac{g^2}{4\pi} \right)^{-1} \sim \left(\frac{\Lambda}{m} \right)^4 \to \infty \,, \tag{89}$$

where Λ is taken as $\Lambda = \Lambda_{\rm TC} = \Lambda_{\rm ETC}$. This is similar to the case i) except that the VM yields $\hat{S} \sim 1/m^4$, while case i) does $\hat{S} \sim \ln(f_\pi/M_\rho) \sim \ln(m/M_\rho)$.

2. Example for Case ii)

The conformal phase transition, corresponding to the case ii), has been indicated by the straightforward calculation of large N_f QCD based on the ladder SD equation and the BS equation [4, 17]. It was shown [17] that in the ladder approximation homogeneous BS equation together with SD equation (for $N_{\rm TC} = 3$ case) gives the bound state mass M_ρ as well as f_π, which scales near the conformal window as

$$\begin{aligned} f_\pi \ &\sim \ 0.375\, m \to 0 \ \ , \quad M_\rho \sim 4.13\, m \to 0 \,, \\ &\frac{f_\pi}{M_\rho} \simeq \text{constant} \simeq 0.091 \,. \end{aligned} \tag{90}$$

Then we have $(f_\pi z_m)^2 \simeq (2.4)^2 (f_\pi/M_\rho)^2 \simeq 0.048$ and hence can read off ξ from Eq.(62) with $\gamma_m \simeq 1$ as (see Fig. 2):

$$\xi^2 \simeq \text{constant} \simeq (1.63)^2 \,, \tag{91}$$

which is $N_{\rm TC}$-independent.

On the other hand, $\hat{S}$ was straightforwardly calculated through current correlators by the ladder SD equation and inhomogeneous BS equation [4]. The result shows that $\hat{S}$ slightly decreases as we approach the conformal phase transition point: $\hat{S} \simeq 0.30$ to $\hat{S} \simeq 0.25$ (for $N_{\rm TC} = 3$):

$$\frac{\hat{S}}{N_{\rm TC}} \simeq 0.10 \to 0.083 \ . \tag{92}$$

However, since the ladder SD and BS method tends to overestimate $\hat{S}$ in QCD, which could be understood as scale ambiguity, the actual value near the conformal phase transition point with $\gamma_m \simeq 1$ should be properly re-scaled by a factor roughly 2/3 to fit the QCD value correctly when the calculation is extended to the QCD case. If this is done, then the value could be

$$\frac{\hat{S}^{\rm (re-scaled)}}{N_{\rm TC}} \simeq 0.067 \to 0.056 \ . \tag{93}$$

Curiously enough, a set of the values of ξ^2 in Eq.(91) and $\hat{S}/N_{\rm TC}$ in Eq.(93) fit in the line of the holographic result in Fig.4. At this moment it is not clear whether or not this coincidence has deeper implications.

3. Example for Case iii)

As for case iii), at this moment we have no concrete dynamics which gives rise to the situation that, near the conformal phase transition point $m \to 0$, we have $\hat{S} \sim \xi^2 \to 0$ in such a way that

$$\xi^2 N_{\rm TC} \sim (f_\pi z_m)^2 \sim \left(\frac{f_\pi}{M_\rho} \right)^2 \to 0 \,, \qquad \text{as} \quad m \to 0 \,. \tag{94}$$

If it is really realized, our holographic result would imply somewhat severe constraint on the value of ξ in Eq.(70): $\xi < 0.59$ for a typical W/C TC with one family techni-fermions ($N_f = 8$) and $N_{\rm TC}$, which actually corresponds to $M_\rho > 3.3$TeV. Although this might confront the perturbative unitarity problem #10, this could be resolved by the techni-dilaton (as a composite Higgs) dynamically formed in the generic W/C TC, which could be identified with the bulk scalar in the present holographic approach.

Such a case may be realized in a bizarre situation that in contrast to the usual picture $m \sim z_m^{-1} \sim M_\rho$, there may be no bound states as a remnant of the conformal window except for the NG boson π and the techni-dilaton which could be the only light spectra reflecting the spontaneous chiral symmetry and conformal symmetry and hence could have arbitrarily small mass compared with our infrared scale $m \ll z_m^{-1}$ near the conformal phase transition point. Although the holography gives an infinite set of bound states consistently with the large N_c limit, the conformal phase transition essentially depends on the Banks-Zaks infrared fixed point which is realized only for large N_f (with $N_f/N_c =$ fixed) instead of the large N_c limit. Thus all the massive bound states would quickly decay into the constituents i.e. pairs of the light techni-fermions (or, π's) through the N_c subleading effects. We should note that such a picture is compared with the explicit computation based on the ladder SD and BS equations [17] which actually produce light bound states of ρ and a_1 but $M_\rho, M_{a_1} > 2m$ in contrast to π (massless) and the scalar meson ("techni-dilaton" with mass $M_{\rm TD} \simeq \sqrt{2}m < 2m$) near the conformal phase transition point. Then the vector and axialvector bound states may in principle quickly decay into pair of the light techni-fermions (or light composite pi's). We will see a definite answer to this possibility by the lattice calculations of the large N_f QCD in near future.

Another possibility to have small $\hat{S}$ would be to include subleading corrections in $1/N_c$ to the holography which is valid only at leading order of $1/N_c$. Recall that $\hat{S}$ is given in terms of vector and axialvector pole contributions as in Eq.(95) . Although the holography includes all the resonance contributions, it is only the result at $1/N_c$ leading order. The subleading effects may change the result drastically particularly near the conformal phase transition point where N_f is large with $N_f/N_c =$ fixed and we are considering corrections to vanishingly small quantities. We can always integrate out higher resonances in the holographic theory to arrive at the (generalized) HLS model which has only few lowest vector and axialvector resonances ("holographic reduction") [31]. Then the loop contributions of this theory yield part of the subleading corrections in $1/N_c$ to the holography [31]. Now look at the generalized HLS model containing only ρ and a_1 [24] as a consequence of the above holographic reduction, which corresponds to taking only contributions from the lowest resonances ρ and a_1:

$$\hat{S} = 4\pi \left(\left(\frac{F_\rho}{M_\rho}\right)^2 - \left(\frac{F_{a_1}}{M_{a_1}}\right)^2 \right) = \left(\frac{g^2}{4\pi}\right)^{-1} \left(1 - \left(\frac{b}{b+c}\right)^2\right) , \tag{95}$$

where b, c are the parameters of this generalized HLS model to be running at loop level, which is compared with the simplest HLS model, Eq.(89). Differing from the VM based on the one-loop calculations of the simplest HLS model having only ρ without a_1 [25, 30], the chiral restoration due to the one loop contributions of this model is more involved [32], which may suggest a possibility of a fixed point of the HLS parameters for giving a vanishing $\hat{S} \sim (f_\pi/M_\rho)^2 \sim c/g^2 \to 0$ due to cancellation among ρ and a_1 contributions at the chiral restoration point.

Finally, we shall emphasize that, in addition to the scaling relation of $\hat{S}$, holographic calculation has provided us with another scaling relation, Eq.(88), that is the scaling relation of f_π with respect to m. Note that Eq.(88) takes a quite different form compared to Eq.(87) (which is the familiar form, $f_\pi^2/N_f \sim m^2$, as seen in QCD even), and hence it could be a key ingredient in a future search for an example of the case iii) a development of lattice calculation for large N_f QCD would clarify whether such a scaling property can actually be realized.

VI. SUMMARY

In this paper we have studied the S parameter in the walking/conformal technicolor (W/C TC), based on the deformation of holographic QCD by varying the anomalous dimension of techni-fermion condensation from the QCD monitor value $\gamma_m \simeq 0$ to that of the W/C TC $\gamma_m \simeq 1$. In contrast to the previous authors who worked on $\gamma_m = 1$ and particular values of ξ, we gave an explicit functional form of $\hat{S}$ in the entire parameter space $0 < \xi < \infty$ and $0 < \gamma_m < 1$, which turned out to be fairly independent of the value of γ_m and to behave as $\hat{S} \sim \xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2$ for $\xi \ll 1$. Thus $\hat{S} \ll 1$ can be realized, if we have a dynamics showing $\xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \ll 1$ near the conformal window where the chiral symmetry get restored $\langle \bar{T}T \rangle \to 0$. However, although ξ is proportional to $\langle \bar{T}T \rangle$, we found

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#10 We thank R.S. Chivukula for useful comments on this point.

no suppression of ξ or f_π/M_ρ and hence of $\hat{S}$ due solely to the large anomalous dimension in contrast to the claim of the literature, through careful analysis of the renormalization-point dependence of the $\langle\bar{T}T\rangle$.

Although the ratio f_π/M_ρ cannot be calculated in the holography, we discussed possible behavior of it near the conformal window where the chiral symmetry gets restored $\langle\bar{T}T\rangle \to 0$. To compare the holographic result to that of more explicit dynamics, we classified holographic W/C TC models into three cases: i) $f_\pi/M_\rho \to \infty$, ii) $f_\pi/M_\rho \to$ constant $\neq 0$, iii) $f_\pi/M_\rho \to 0$.

Case i) roughly corresponds to the Vector Manifestation (VM) [25] realized in the simplest HLS model with ρ and π , which yields $\hat{S} \to \infty$.

Case ii) corresponds to the result of the ladder SD and BS equation [4, 30] , which to our surprise yields not only $\xi^2 N_{\rm TC} \sim (f_\pi/M_\rho)^2 \to$ constant but also a set of the calculated values of f_π/M_ρ and $\hat{S}$ well fit to the line of the parameter space of the holographic result in this paper. Deeper implications of this coincidence are not clear at this moment.

Although Case iii) has no explicit dynamics at the moment, if it is realized, the holographic result we obtained seems to pose a severe constraint on the lower bound of techni-ρ mass. We discussed a possibility for having such a case where there are no bound states as a remnant of the conformal window except for the NG boson π and the scalar (as a techni-dilaton), and hence $m \ll z_m^{-1}$. Actually, the dynamics near the conformal phase transition is governed by the Banks-Zaks infrared fixed point due essentially to the large N_f with $N_f/N_c =$ fixed but not to the simple large N_c limit. We also discussed another possibility for having $\hat{S} \ll 1$ by introducing $1/N_c$ subleading corrections through the meson loops in the generalized HLS model. Note that the generalized HLS model is obtained by the integrating out the higher resonances of the holographic result which is valid only at $1/N_c$ leading order. We also found that if the case iii) is realized in some concrete dynamics, we have a novel scaling property of f_π with respect to m (Eq.(88)), which takes a quite different form compared to the familiar form, $f_\pi^2/N_{\rm TC} \sim m^2$. This would suggest that this scaling property may play an important role to reveal such a phenomenologically viable W/C TC.

For all these unsolved features, the holographic relation we obtained in this paper would be useful for further studies of the W/C TC.

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APPENDIX A: LIMIT OF $\gamma_m \to 1$

We shall discuss in this Appendix on the limit $\gamma_m \to 1$ of our result which we show can be continuously moved over to $\gamma_m = 1$.

Let us begin with the classical solution of $v(z)$ of Eq.(18) for $\gamma_m = 1$, which is given by

$$v(z)^{(\epsilon)} = C_1 \left(\frac{z}{L}\right)^2 + C_2 \left(\frac{z}{L}\right)^2 \ln \frac{z}{L}. \tag{A.1}$$

On the other hand, Eq.(22) for $\gamma_m < 1$ takes the form in the limit $\gamma_m \to 1$:

$$\begin{aligned} v(z)^{(\epsilon)} &= \left(\frac{z}{L}\right)^2 \left(c_1 \left(\frac{z}{L}\right)^{-\delta} + c_2 \left(\frac{z}{L}\right)^{\delta}\right) \\ &\overset{\delta \ll 1}{\simeq} \left(\frac{z}{L}\right)^2 \left((c_1 + c_2) + (c_2 - c_1)\delta \cdot \ln \left(\frac{z}{L}\right)\right) , \end{aligned} \tag{A.2}$$

where $\delta \equiv 1 - \gamma_m$.

Comparing Eqs.(A.1) and (A.2), we read C_1 and C_2 as

$$\begin{aligned}
C_1 &= \lim_{\delta\to 0}(c_1+c_2) = \lim_{\delta\to 0}\left[\left(\frac{L}{z_m}\right)^2\frac{\xi}{L} + \frac{\left(1-\left(\frac{L}{z_m}\right)^{2-2\gamma_m}\right)}{\left(1-\left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m}\right)}\left(\mathcal{M}-\left(\frac{L}{z_m}\right)^2\frac{\xi}{L}\right)\right] \\
&= \left(\frac{L}{z_m}\right)^2\frac{\xi}{L} + \frac{\ln z_m/L}{\ln z_m/\epsilon}\left(\mathcal{M}-\left(\frac{L}{z_m}\right)^2\frac{\xi}{L}\right), \qquad \text{(A.3)} \\
C_2 &= \lim_{\delta\to 0}[(c_2-c_1)\delta] = \lim_{\delta\to 0}\left[\left(\left(\frac{L}{z_m}\right)^2\frac{\xi}{L} - 2\frac{\left(\mathcal{M}-\left(\frac{L}{z_m}\right)^2\frac{\xi}{L}\right)}{1-\left(\frac{\epsilon}{z_m}\right)^{2-2\gamma_m}}\right)\delta\right] \\
&= -\frac{1}{\ln\frac{z_m}{\epsilon}}\left(\mathcal{M}-\left(\frac{L}{z_m}\right)^2\frac{\xi}{L}\right), \qquad \text{(A.4)}
\end{aligned}$$

We then find

$$v(z)^{(\epsilon)} = C_1\left(\frac{z}{L}\right)^2 + C_2\left(\frac{z}{L}\right)^2\ln\frac{z}{L} = \left(\frac{z}{L}\right)^2\left[\frac{\ln\frac{z}{\epsilon}}{\ln\frac{z_m}{\epsilon}}\left(\frac{L}{z_m}\right)^2\frac{\xi}{L} + \frac{\ln\frac{z}{z_m}}{\ln\frac{\epsilon}{z_m}}\mathcal{M}\right], \qquad \text{(A.5)}$$

which obviously satisfies the boundary conditions $v(\epsilon)^{(\epsilon)} = \left(\frac{\epsilon}{L}\right)^2\mathcal{M}$ and $v(z_m)^{(\epsilon)} = \frac{\xi}{L}$ as it should. Now we may define the current mass M as

$$M \equiv \left[\left(\frac{L}{\epsilon}\right)^2\frac{1}{\ln\left(\frac{z_m^2}{\epsilon^2}\right)}\right]v(\epsilon)^{(\epsilon)} = \mathcal{M}\left[\frac{1}{\ln\left(\frac{z_m^2}{\epsilon^2}\right)}\right], \qquad \text{(A.6)}$$

which yields

$$\begin{aligned}
v(z)^{(\epsilon)} &= \left(\frac{z}{L}\right)^2\left[\frac{\ln\frac{z}{\epsilon}}{\ln\frac{z_m}{\epsilon}}\left(\frac{L}{z_m}\right)^2\frac{\xi}{L} - 2M\ln\left(\frac{z}{z_m}\right)\right] \\
&= \left(\frac{z}{L}\right)^2\left[\left(\frac{\ln\frac{z}{\epsilon}}{\ln\frac{z_m}{\epsilon}}\left(\frac{L}{z_m}\right)^2\frac{\xi}{L} + M\ln\left(\frac{z_m^2}{L^2}\right)\right) - 2M\ln\frac{z}{L}\right]. \qquad \text{(A.7)}
\end{aligned}$$

When we take the limit $\epsilon(< z)\to 0$, we have

$$v(z)^{(\epsilon)} \quad \xrightarrow{\epsilon\to 0} \quad v(z) = \left(\frac{z}{L}\right)^2\left[\left(\left(\frac{L}{z_m}\right)^2\frac{\xi}{L} + M\ln\left(\frac{z_m^2}{L^2}\right)\right) - 2M\ln\frac{z}{L}\right]. \qquad \text{(A.8)}$$

This takes the form in the chiral limit $M = 0$:

$$v(z) = \left[\left(\frac{L}{z_m}\right)^2\frac{\xi}{L}\right]\cdot\left(\frac{z}{L}\right)^2, \qquad \text{(A.9)}$$

which is in accord with the $\gamma_m < 1$ case, Eqs.(22) and (25) for $M = 0$.

The technifermion condensate is given by Eq.(14) as

$$\langle\bar{T}T\rangle = \left(\frac{z_m}{L}\right)\cdot\left[-\frac{2}{z_m^3}\frac{L}{g_5^2}\xi\right] = Z_m^{-1}(x)\Big|_{x=\frac{1/L}{1/z_m}}\cdot\langle\bar{T}T\rangle_{1/z_m}, \qquad \text{(A.10)}$$

where

$$Z_m^{-1}(x) = x, \quad \langle\bar{T}T\rangle_{1/z_m} = -\frac{2}{z_m^3}\frac{L}{g_5^2}\xi = -\frac{N_{TC}}{6\pi^2}\frac{1}{z_m^3}\xi. \qquad \text{(A.11)}$$

Then it is clear that $\langle\bar{T}T\rangle$ is the quantity renormalized at $1/L$: $\langle\bar{T}T\rangle = \langle\bar{T}T\rangle_{1/L}$ and that ξ does not depend on $1/L$ which corresponds to the renormalization point. We may introduce the bare mass and bare condensate as

$$\begin{aligned} \langle\bar{T}T\rangle_0 &= \langle\bar{T}T\rangle_{1/\epsilon} = Z_m^{-1}(L/\epsilon)\cdot\langle\bar{T}T\rangle = Z_m^{-1}(z_m/\epsilon)\cdot\langle\bar{T}T\rangle_{1/z_m}\,, \\ M_0 &= Z_m(L/\epsilon)\cdot M\,, \end{aligned} \tag{A.12}$$

so that the multiplicative renormalization of mass operator is evident:

$$M_0\langle\bar{T}T\rangle_0 = M\langle\bar{T}T\rangle\,. \tag{A.13}$$

The anomalous dimension is thus given as

$$\gamma_m = \left.\frac{\partial \ln Z_m^{-1}(x)}{\partial \ln x}\right|_{x=L/\epsilon} = 1 \tag{A.14}$$

in agreement with our procedure in the text taking the limit $\gamma_m(<1)\to 1$.

APPENDIX B: RELATIONSHIP BETWEEN f_π AND m IN HOLOGRAPHY

In this section, we compare the holographic expression of f_π^2 in terms of m as given in Eqs.(87) and (88) with the Pagels-Stokar formula [33] for f_π^2.

The Pagels-Stokar formula [33] relates the pion decay constant f_π to a mass function of techni-fermion $\Sigma(x)$ with $x = -p^2$ as

$$\frac{(4\pi^2)}{N_c}\cdot f_\pi^2 = \int_{\mathrm{IR}}^{\mathrm{UV}} xdx\frac{\Sigma(x)(\Sigma(x) - x\Sigma'(x)/2)}{(x+\Sigma^2(x))^2}\,, \tag{B.1}$$

where we have introduced "IR" and "UV" cutoffs in integral with respect to x. It should be noted that, for regions (i) $x > m^2$ and (ii) $x < m^2$, the mass function $\Sigma(x)$ can be expressed in terms of the dynamical fermion mass $m \equiv \Sigma(m^2)$ with the anomalous dimension γ_m $(0 < \gamma_m < 1)$ as

$$\text{(i)}\, x > m^2, \qquad \Sigma_{(i)}(x) \sim \frac{m^3}{x}\left(\frac{x}{m^2}\right)^{\gamma_m/2}\,, \tag{B.2}$$

$$\text{(ii)}\, x < m^2, \qquad \Sigma_{(ii)}(x) \sim m\,. \tag{B.3}$$

To make contact with holographic calculations, we may identify IR and UV scales as

$$\mathrm{UV} \equiv (\epsilon^{-1})^2 = \Lambda^2\,, \qquad \mathrm{IR} \equiv (z_m^{-1})^2 \simeq M_\rho^2\,. \tag{B.4}$$

Note that the integration in x necessarily results in convergence as for $0 < \gamma_m < 1$. Therefore we hereafter evaluate f_π taking the continuum limit $\Lambda\to\infty$, but keep dependence of the IR scale M_ρ in the expression of f_π.

Let us first examine the case that $m > M_\rho$. Putting the asymptotic expression for $\Sigma(x)$ given in Eqs.(B.2) and (B.3) into Eq.(B.1), we straightforwardly calculate the right hand side of Eq.(B.1) as

$$\begin{aligned} \frac{(4\pi^2)}{N_c}\cdot f_\pi^2(m > M_\rho) &= \int_{M_\rho^2}^{m^2} xdx\frac{\Sigma_{(ii)}(x)(\Sigma_{(ii)}(x) - x\Sigma'_{(ii)}(x)/2)}{(x+\Sigma^2_{(ii)}(x))^2} \\ &\quad + \int_{m^2}^{\infty} xdx\frac{\Sigma_{(i)}(x)(\Sigma_{(i)}(x) - x\Sigma'_{(i)}(x)/2)}{(x+\Sigma^2_{(i)}(x))^2} \\ &= \int_{M_\rho^2}^{m^2} xdx\frac{m^2}{(x+m^2)^2} \\ &\quad + \int_{m^2}^{\infty} xdx\frac{\Sigma_{(i)}(x)(\Sigma_{(i)}(x) - x\Sigma'_{(i)}(x)/2)}{(x+\Sigma^2_{(i)}(x))^2} \\ &= m^2\left[\ln 2 - \frac{1}{2} + \left(\frac{3+\Delta}{8\Delta^2}\right)\left(\Delta - \psi(\frac{1}{2} - \frac{1}{2\Delta}) + \psi(1-\frac{1}{2\Delta})\right)\right] \\ &\quad + \mathcal{O}\left(\left(\frac{M_\rho}{m}\right)^4\right)\,, \end{aligned} \tag{B.5}$$

where ψ denotes a poli-gamma function. From Eq.(B.5), we see that f_π scales as $m \to 0$ independently of IR cutoff scale M_ρ for any value of γ_m,

$$f_\pi^2(m > M_\rho) \sim m^2 \,. \tag{B.6}$$

which results in the same form as that of Eq.(87).

We next turn to the case $m \ll M_\rho$. By putting Eq.(B.2) into Eq.(B.1), f_π may be calculated to be

$$\begin{aligned} \frac{(4\pi^2)}{N_c} \cdot f_\pi^2(m \ll M_\rho) &= \int_{M_\rho^2}^{\infty} x dx \frac{\Sigma_{(i)}(x)(\Sigma_{(i)}(x) - x\Sigma'_{(i)}(x)/2)}{(x + \Sigma_{(i)}^2(x))^2} \\ &= \frac{m^2}{2-\gamma_m} \cdot \left(\frac{m}{M_\rho}\right)^{2\gamma_m - 4} \\ &\quad \times F\left[2, \frac{2-\gamma_m}{3-\gamma_m}, \frac{5-2\gamma_m}{3-\gamma_m}, -\left(\frac{m}{M_\rho}\right)^{6-2\gamma_m}\right] , \end{aligned} \tag{B.7}$$

where F denote a hyper-geometric function. From Eq.(B.7), in the limit $m/M_\rho \to 0$, we find a scaling relation sensitive to both M_ρ and m,

$$f_\pi^2(m \ll M_\rho) \sim m^2 \cdot \left(\frac{m}{M_\rho}\right)^{4-2\gamma_m} \quad , \tag{B.8}$$

which is in accord with that of Eq.(88).

Thus it turn out from Eqs.(B.6) and (B.8) that the scaling relations (87) and (88) between f_π and m, calculated in holographic technicolors, are exactly reproduced by the PS formula.

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